Golden Ratio Sidebands

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Abstract

The Golden Ratio has unique mathematical properties which have important physical meaning when applied to the nonlinear mixing of Electromagnetic waves. Consider two waves, whose frequencies are in the Golden Ratio to each other. If these waves heterodyne, the resulting Sidebands will form a Geometric Series with the original two waves. Heterodyning any two consecutive terms in this series will produce the next and previous terms.

1. Introduction

Consider two waves, with frequencies f and αf, where α is some ratio.

$$f = Fundamental$$
$$\alpha = Real\ Number > 1$$

If these two waves Heterodyne, we will get two Sidebands: The Lower and Upper Sidebands.

$$\alpha f - f \quad \text{(Lower Sideband)}$$
$$\alpha f + f \quad \text{(Upper Sideband)}$$

Let's plot all four waves in the frequency domain.

$$\alpha f - f = A$$
$$f = B$$
$$\alpha f = C$$
$$\alpha f + f = D$$

It is possible to choose a value for α, such that:

$$\frac{B}{A} = \frac{C}{B} = \frac{D}{C} \quad (1)$$

This value for α would be a solution to the equation:

$$X^2 - X - 1 = 0 \quad (2)$$

The Positive Root is:

$$\alpha = \frac{1+\sqrt{5}}{2} = 1.618033... \quad (3)$$

Euclid called this the Extreme and Mean Ratio [1]. It is also called the Golden Ratio.

2. Properties of the Golden Ratio

It is a property of the Golden Ratio that:

$$\alpha^n + \alpha^{n+1} = \alpha^{n+2} \quad (4)$$
$$\alpha^n - \alpha^{n-1} = \alpha^{n-2} \quad (5)$$

Therefore, it is useful to use this number as a base of logarithms.

Consider the series:

$$..., f\alpha^{-3}, f\alpha^{-2}, f\alpha^{-1}, f\alpha^{0}, f\alpha^{1}, f\alpha^{2}, f\alpha^{3}, ... \quad (6)$$

The sum of any two consecutive terms is equal to the following term. The difference between two consecutive terms is equal to the previous term.

3. Heterodyne Example

We begin with two frequencies, f and αf, which are two consecutive terms in the Geometric Series above. For example:

$$f = f\alpha^0 \quad (7)$$
$$\alpha f = f\alpha^1 \quad (8)$$

When f and αf heterodyne, we get these two Sidebands:

$$\alpha f - f = f\alpha^1 - f\alpha^0 = f\alpha^{-1} \quad \text{(LSB)}$$
$$\alpha f + f = f\alpha^1 + f\alpha^0 = f\alpha^{2} \quad \text{(USB)}$$

Which are the previous and next terms in the series.

4. Second Order Products

If the Lower Sideband heterodynes with f, we get:

$$f\alpha^0 + f\alpha^{-1} = f\alpha^1 \quad (9)$$
$$f\alpha^0 - f\alpha^{-1} = f\alpha^{-2} \quad (10)$$

It reinforces the frequency at αf, and also gives us a new term in the series. This new term has a lower frequency than the Lower Sideband.

If the Upper Sideband heterdynes with αf, we get a similar phenomenon. The new wave will have a higher frequency than the upper sideband.

5. Feedback and Recursion

Consider a system in which the Sidebands are fed back into the nonlinear element which causes the waves to heterodyne. This will produce additional terms in the Geometric Series.

6. Conclusion

Antennas or other resonant structures, whose physical geometry is based on the Golden Ratio, may preform well in Spread Spectrum applications.

More specifically, they may be useful for generating and receiving sets of frequencies which form a Geometric Series in the frequency domain.

References

[1] Euclid. Elements. Book VI, Definition 3. 1